# Negative-result experiments, and the requirement of wavefunction collapse



1992 J. Phys. A: Math. Gen. 25 2387

(http://iopscience.iop.org/0305-4470/25/8/045)

J. Phys. A: Math. Gen. **25** (1992) 2387–2394. Printed in the UK

# Negative-result experiments, and the requirement of wavefunction collapse

D Home† and M A B Whitaker

Department of Pure and Applied Physics, The Queen's University of Belfast, Belfast BT7 1NN, UK

Received 12 November 1991

**Abstract.** We give a general analysis of negative-result experiments, and argue that they do not require the hypothesis of wavefunction collapse, as suggested by other workers. We point out that the regeneration phenomenon involving neutral kaons provides an experimentally tested example of this type of measurement. We also show that the phenomenon of interrupted fluorescence in the V-configuration of atomic physics may be explained in a straightforward way without the use of collapse.

## 1. Introduction

Negative-result experiments in quantum theory have been discussed for a considerable time (Renninger 1953, 1960, Epstein 1945, Jammer 1974). In such experiments, the result is obtained not through the occurrence of a physical event, as would be the case for a normal measurement, but by the absence of such an event.

Several different types of such experiment have been discussed. In that considered by Dicke (1981, 1986), failure of a particle to scatter photons from a light beam may be said to demonstrate that the particle lies outside the beam.

In interrupted fluorescence experiments on the so-called V-configuration of atomic physics (Dehmelt 1975, Cook and Kimble 1985, Schenzle and Brewer 1986, Porrati and Putterman 1987, Pegg and Knight 1988a, b, Pegg 1991), failure of a single atom to make transitions from a highly fluorescent state $|3\rangle$ to the ground state $|1\rangle$, may be said to tell us that the atom is 'shelved' in another level $|2\rangle$ which decays to the ground state far more slowly than $|3\rangle$. (In these experiments, the $|1\rangle$–$|3\rangle$ transition is usually strongly driven, and the $|1\rangle$–$|2\rangle$ transition weakly driven.)

In the type of experiment used in analysis of the quantum Zeno effect (Chiu *et al* 1977, Peres 1980, Home and Whitaker 1986), failure to detect a decay particle from an unstable atom leads us to deduce that the atom has survived. (Naturally, for this conclusion to be drawn, detectors must totally surround the atom in question.)

In this paper we also discuss the regeneration phenomenon involving neutral kaons (Gell-Mann and Pais 1955, Feynman *et al* 1965), and also a related thought-experiment suggested by Scully *et al* (1978), involving a Stern–Gerlach apparatus with a molecular beam and an atomic detector. Both these experiments may be viewed as negative-result measurements.

† On leave from: Department of Physics, Bose Institute, Calcutta 700009, India.

0305-4470/92/082387+08$04.50 

In section 2 of this paper, we give a general discussion of negative-result experiments, especially in relation to the collapse of the state vector. We argue that it is not necessary to assume collapse for an understanding of these experiments. In section 3, we discuss the kaon regeneration phenomenon, and the related Scully analysis, and in section 4 we discuss the interrupted fluorescence experiments.

Since quantum Zeno processes have been given much attention recently, and indeed have been analysed by Petrovsky *et al* (1990) using much the same ideas as those of this paper, this effect is not discussed fully here, but we frequently refer to it to demonstrate the connections with the present work.

## 2. Negative-result experiments and state-vector collapse

The main conceptual problem with negative-result experiments is, of course, that prior to the experiment, the system is supposed, quantum-mechanically, to be in a linear combination of states corresponding to the positive and negative results. In the Dicke case, for example, the linear combination is of states where the particle lies inside and outside the beam. The fact of obtaining the negative result appears enough to change the state of the particle to one where it lies outside the beam, without any obvious interaction between photon and particle.

In the quantum Zeno case, if no observation were carried out, one would expect the decaying atom to be in a linear combination of surviving and decayed states. If no decay particle is observed, this appears to indicate that the atom has actually remained in the surviving state, although it seems that no interaction has taken place between atom and observing apparatus.

The important achievement of Dicke (1981, 1986) was to indicate the limitations of this point of view for the type of experiment he considers. He showed it is possible to consider that the unscattered photon involved in the negative-result experiment has in fact been absorbed and re-emitted by the atom. He was also able to show, contrary to one's initial suspicions, that the process does not entail non-conservation of energy or momentum. For the experiment to be possible, the photon must initially be in a state with neither momentum nor energy well defined; thus it may transfer energy and/or momentum to or from the atom but overall remain unchanged itself.

Dicke insists—somewhat more explicitly in his 1986 paper than in 1981—on a contraction of the wavepacket taking place at the measurement. He writes of an 'irreversible decision process' being represented by a projection operator inducing a change of wavefunction incompatible with the Schrödinger equation. Several of those discussing the interrupted fluorescence experiments (Porrati and Putterman 1987, Pegg and Knight 1988a, b, Pegg 1991) also make central use of the collapse concept.

Yet discussion of wavefunction collapse is highly controversial within quantum measurement theory. Throughout this paper we mean by the term 'collapse' that the combined state vector of observed system and macroscopic measuring apparatus actually becomes a mixed state by the conclusion of the measurement (von Neumann 1955). Explicitly we may say that during the measurement process, the states of observed and observing systems couple to a form $\Sigma_n c_n|\phi_n\rangle|\psi_n\rangle$, where the $|\phi_n\rangle$ represent observed system, and the $|\psi_n\rangle$ the apparatus. Collapse, if it takes place, replaces the superposition by a mixture.

As already stated, the collapse process cannot be reconciled with orthodox quantum theory, and so the collapse argument implies that systems behave differently under 'measurement' than in other situations. Yet, as has been particularly emphasized by Bell (1981), the term 'measurement' is not fundamental; one would expect that any measurement process could be described in terms of primitive interactions, which presumably should obey the Schrödinger equation.

It may also be mentioned that some physicists, particularly supporters of ensemble interpretations such as Ballentine (1970), believe that it is possible to explain the quantum measurement phenomenon without the idea of collapse. Their argument is essentially that the orthogonality of the $|\psi_n\rangle$ can provide many of the results normally thought to require collapse. In particular, it can ensure that (immediately) repeated measurement gives the same answer as the first measurement. From the point of view of the $|\phi_n\rangle$ alone, the linear combination above will behave as a mixture; the presence of the orthogonal $|\psi_n\rangle$ means that the reduced density matrix (involving the $|\phi_n\rangle$ only) is diagonal. The superposition above may be regarded in this way as an 'improper mixture' (d'Espagnat 1976). (It may also be noted that the difficulty of the above argument is the assumption that the state of a macroscopic apparatus may be a linear combination of states $|\psi_n\rangle$.)

Because of this controversy, it is particularly important to ascertain whether the negative-result experiments discussed in this paper really require the collapse hypothesis. If they do require it, that would be an important discovery. If, however, they do not require it (as distinct from its use being possibly a convenience), and we shall argue that this is the case, we should certainly not claim to use the experiments to prove the existence of collapse.

We may, indeed, take the preceding discussion one stage further. The same argument will apply if the $|\psi_n\rangle$ in the above superposition represent not macroscopic apparatus states, but orthogonal states of some microscopic system with which the $|\phi_n\rangle$ have become entangled. In this case, of course, the microscopic nature of the $|\psi_n\rangle$ would make any idea of wavefunction collapse entirely inappropriate, but their presence and orthogonality can still yield the same type of experimental result. (It may also be noted that the difficulty mentioned above for the case where the $|\psi_n\rangle$ are macroscopic does not apply in this case.)

A good example of the above was the paper by Itano *et al* (1990a). These authors produced extremely interesting experimental results which, they claimed, demonstrated the quantum Zeno effect and wavefunction collapse. Several authors (Petrovsky *et al* 1990, Peres and Ron 1990, Ballentine 1991) were able to show that the results could be explained using only the Schrödinger equation, and so certainly could not be a proof of collapse. The appropriate $|\psi_n\rangle$ in this case are microscopic states of emitted photons. (For further discussion of the requirement for collapse in quantum Zeno processes, see Whitaker 1989, Home and Whitaker 1992a, b.)

We note that Itano *et al* (1990b) have replied by suggesting that the term 'wavefunction collapse' may be extended to the cases just considered. However, we consider that to call by the same name (i) interaction with a macroscopic apparatus in which a mixture is formed, and (ii) interaction with a microscopic system where a superposition is maintained, is unhelpful.

## 3. Regeneration of neutral kaons

We now turn to the regeneration of neutral kaons (Gell-Mann and Pais 1955). If a beam of neutral kaons in the pure state, $|K^0\rangle$, is allowed to travel in vacuum for a time sufficiently longer than the $K_s$ lifetime (about $10^{-10}$ s), one is left with kaons in the pure state $|K_L\rangle$, where

$$|K_L\rangle = (1/\sqrt{2})(|K^0\rangle + |\bar{K}^0\rangle). \quad (1)$$

If this beam of kaons interacts with a thick slab of material, the $|\bar{K}^0\rangle$ component interacts strongly with the nucleons giving rise to $\Lambda$ and $\Sigma$, while the $|K^0\rangle$ component is predominantly elastically scattered.

The final state of the kaons after interaction with the nucleons can be symbolically written as

$$|\psi_f\rangle = (1/\sqrt{2})(|K^0\rangle|A_0\rangle + |\bar{K}^0\rangle|A_+\rangle). \quad (2)$$

Here $|A_0\rangle$ and $|A_+\rangle$ correspond to unexcited and excited states of the nucleons respectively, with $\langle A_0|A_+\rangle = 0$. The $|\bar{K}^0\rangle|A_+\rangle$ component is, so to say, 'annihilated' by being registered as $\Lambda$ and $\Sigma$ particles in appropriate detectors within the slab of matter. Thus, for this component, the experiment is a 'positive result' one.

The other component emerges from the slab, and causes no registrations in the detectors. This lack of registrations indicates that one is dealing with the $|K^0\rangle|A_0\rangle$ component, and that any subsequent detection must yield a $|K_0\rangle$. Yet the passage through the slab corresponds to a 'negative result' experiment for this component. Apparently the kaons involved are not subject to any local interaction involving an exchange of energy or momentum. Yet the simplest description would seem to be that, under this lack of interaction, the $|K_L\rangle$ has changed to a $|K^0\rangle$. Since

$$|K^0\rangle = (1/\sqrt{2})(|K_L\rangle + |K_s\rangle) \quad (3)$$

the emergent particles can decay in either $K_L$ or $K_s$ mode, while the incident ones were restricted to the former mode.

The change in state and property can be understood using a 'realist' interpretation of quantum mechanics (in which a wavefunction is taken to be an objectively real entity describing the state of an individual particle). One would argue that 'interaction' of the nucleons occurs with the pure state (given by equation (1)) of a single kaon resulting in the entangled state vector $|\psi_f\rangle$ involving correlation between the kaon states and the states of the nucleons. It is this 'interaction' which leads to a new state vector for an emergent kaon.

A formal way of providing a causal space–time description of this type of 'interaction' is through the quantum-potential approach (Bohm and Hiley 1987). However, unless the physical origin of the quantum potential can be clarified, a detailed understanding of such an apparently peculiar 'interaction' and related questions such as those concerning energy and momentum conservation at the level of individual particles will remain unclear.

It is stressed that the foregoing analysis cannot be interpreted as verifying wavefunction collapse. To see this we note that testing collapse would mean discriminating between the pure state given by equation (2), and the mixed state comprising $|K^0\rangle|A_0\rangle$ and $|\bar{K}^0\rangle|A_+\rangle$. The coupling of the $|K^0\rangle$ to the $|A_0\rangle$, the $|\bar{K}^0\rangle$ to the $|A_+\rangle$ ensures that $|K^0\rangle$ particles will appear in an emergent beam. It is certainly not necessary to assume a mixed state. The assumption of collapse may seem to provide a convenient way of understanding such examples but we should be careful to distinguish between a matter of convenience, and the necessity of such an assumption in accounting for the empirically verified facts.

We now briefly refer to the work of Scully *et al* (1978). The kaon regeneration experiment just discussed may, in fact, be viewed as a realization of this type of thought experiment. Scully *et al* use a Stern–Gerlach apparatus with a molecular beam and an atomic detector. In an analogous way to our equation (2), they produce a final state vector which is a superposition of coupled states of beam and (atomic) detector. Such a state vector does describe several features of the expected experimental results; in particular the reduced density matrix for the molecular beam is diagonal.

However, we do not agree with them that the analysis describes collapse, or, as they call it, 'state reduction'. To explain the expected results, one only requires that the states of the interacting systems are entangled, as in equation (2), together with the orthogonality of the (atomic) detector states. One does not require the joint pure state of molecular beam and atomic detector actually to collapse to a mixed state.

## 4. Interrupted fluorescence in the atomic V-configuration

We now turn to the observation of interrupted fluorescence using the atomic V-configuration. Light may be emitted as a series of bright periods of rapid photon emission. For the case of incoherent irradiation, Cook and Kimble (1985) were able to give a simple description of the process, assuming from the outset that sudden jumps between bright and dark periods do occur.

However, for the case of coherent illumination, the situation appears less clear-cut. The intuitive view, in which an atom can be found at a particular time in only one of its three eigenstates, would still predict occasional 'shelving' of the atom at level $|2\rangle$, and hence the existence of long dark periods. Empirically this view turns out to be correct.

However, a more apparently sophisticated view suggests that coherent superposition of states must occur, so that, loosely speaking, the electron occupies all levels 'simultaneously'. The density matrix would thus contain off-diagonal as well as diagonal elements; while the diagonal elements would give probabilities of occupation of the different levels, the off-diagonal elements would provide information about phase coherence. This is pointed out by Schenzle and Brewer (1986), who comment that one is 'tempted' to assume, incorrectly, that fluorescence is continuous in time, but reduced in intensity by the presence of the meta-stable state $|2\rangle$.

These authors resolve the dilemma by using a completely different theoretical technique—that of calculation of photon-counting statistics—which gives the 'intuitive' rather than the 'sophisticated' answer.

This work is not, perhaps, physically very accessible, and several authors claim to have resolved the problem in a more straightforward way via the introduction of wavefunction collapse. Pegg and Knight (1988a) define a time interval, $\Delta t$, and state that, if at least one photon corresponding to the $|3\rangle$–$|1\rangle$ transition is detected during $\Delta t$, the wavefunction will collapse so that $\rho_{22}$ is zero at the end of $\Delta t$; if, on the other hand, no photon is detected, collapse takes place so that $\rho_{22}$ is unity at the end of $\Delta t$. At this time, then, the off-diagonal elements $\rho_{12}$, $\rho_{21}$, $\rho_{23}$ and $\rho_{32}$ are also zero. Thus at this time one may say that the system is either 'in' $|2\rangle$, or 'in' a superposition of $|1\rangle$ and $|3\rangle$. The state of an ensemble of systems is a mixture of $|2\rangle$, and a linear combination of $|1\rangle$ and $|3\rangle$. With this strategy, the occurrence of dark periods is, at least, highly plausible, and may be confirmed by direct calculation.

Pegg and Knight (1988a) scarcely attempt to justify their approach beyond claiming agreement with more complicated theoretical methods (that of Schenzle and Brewer), and the relevant experiments. Later (Pegg and Knight 1988b), they state that collapse of wavefunction by a measurement is an accepted principle of quantum mechanics, and Porrati and Putterman (1987) say much the same. We have already stated in section 2 that it is, in fact, a controversial and debated principle. One should avoid it if possible; certainly one should not claim to have demonstrated it experimentally if the relevant experiments may be explained without its use, and without departing from the Schrödinger equation. Here we seek to show that they may be so explained, and therefore it should be avoided.

One may also question whether measurement *per se* is really involved in the process these authors discuss. Of course the photons produced in the decays are available for observation, and indeed such observation is the point of the experiment. Similarly, failure to observe a photon counts as a negative measurement. Nevertheless, it perhaps hardly seems feasible that the behaviour of the atomic system is driven by detection of the photons, conceivably long distances away from the system itself. (In principle, with detectors far enough away, the actual detection could be done after the apparatus containing the atomic system has been dismantled.)

It must be remembered that any position which says that the results of an experiment depend on the participation of the observer, whether in collapse itself, or merely in the entanglement of system and apparatus states, must admit that the system would behave in an entirely different fashion if unobserved. Such a position cannot be proved to be wrong. Indeed, one must be extremely wary of arguments against it; in particular, the argument for the quantum Zeno position exploits just such a position. But certainly one would wish to avoid explicit participation of the measuring device if possible, and we claim that, in the V-configuration experiments, it is indeed possible.

Our description takes specific account of the photons produced in the decays, and in particular of the correlations between the atomic and photon states (as discussed in section 2). It is well known (Petrovsky *et al* 1990) that states with different numbers of photons are orthogonal to each other. From now on we utilize a new notation for the state vector where the second figure denotes the number of photons emitted. We write the state vector at time $t$ as

$$|\psi\rangle(t)=\sum_{p,n} c_{pn}(t)|p,n\rangle. \tag{4}$$

Thus one may imagine that, for the smallest times of the experiment (less than the lifetime of $|3\rangle$), $\tau_3$, the only non-zero coefficients will be $c_{10}$, $c_{20}$ and $c_{30}$. For rather greater times, though, $c_{11}$ and then $c_{31}$ must increase in prominence, corresponding to the high probability of a decay from level $|3\rangle$ to level $|1\rangle$, the accompanying emission of a photon, and the driving of the $|1\rangle$–$|3\rangle$ transition.

So, for a period, the largest values of $c_{3n}$ and $c_{1l}$ will be for $n$ and $l$ equal to 1, but that of $c_{2m}$ for $m$ equal to 0. (Of course $|1,1\rangle$ will gradually give rise to non-zero $c_{21}$, but this quantity will be smaller than $c_{20}$.)

If one now moves on to a time comparable with the lifetime of levels $|2\rangle$, $\tau_2$, one finds that $c_{3n}$ and $c_{1l}$ will be reasonably large where $n$ and $l$ take a range of values in the vicinity of $\tau_2/\tau_3$. However, $c_{2m}$ will be non-zero for all values of $m$ up to around $\tau_2/\tau_3$, but biased to the lower values. Thus there is very little common representation of $|3,n\rangle$ and $|2,m\rangle$ with $n=m$, or $|1,l\rangle$ and $|2,m\rangle$ with $l=m$.

A full density matrix description involving combined atomic and photon states would thus show correlations between these states. The full density matrix must contain off-diagonal as well as diagonal elements; it is idempotent and represents a pure state. However, it follows from our previous remarks that the reduced density matrix, $\rho_r$, considering only atomic states, will have only very small off-diagonal elements between $|2\rangle$ and either $|1\rangle$ or $|3\rangle$.

For longer times, $c_{3n}$, $c_{2m}$ and $c_{1l}$ appear in the state vector with increasing values of $n$, $m$ and $l$. However, $n$ and $l$ will always predominantly take a range of values rather greater than the most common values of $m$. Thus the reduced density matrix will still have very small elements connecting level $|2\rangle$ with the other levels. This implies that we do not have a linear combination of level $|2\rangle$ with either of the other levels. Rather, from the point of the reduced density matrix, the system behaves like a mixture of level $|2\rangle$, and a superposition of levels $|1\rangle$ and $|3\rangle$. Since it is a single atomic system, this implies that it is either 'in' level $|2\rangle$, or 'in' a superposition of $|1\rangle$ and $|3\rangle$, precisely the picture for interrupted fluorescence.

While our analysis so far has been for the full density matrix, examining all possibilities of behaviour, it is interesting to follow through the behaviour of an individual atomic system. In this case it is clear that the analysis reproduces the results obtained by wavefunction collapse. Every time a photon is produced as the atomic system decays from $|3\rangle$ to $|1\rangle$, the levels $|1\rangle$ and subsequently $|3\rangle$ are 'cut off' from $|2\rangle$, not by an extra-quantum-mechanical 'process', but by the presence of the additional photon in the state vector. In similar fashion, the analysis of Petrovsky *et al* (1990), taking regard of photon states, is able to reproduce the results obtained by the assumption of collapse by Itano *et al* (1990a) in their analysis of their quantum Zeno results.

In neither case can the analysis be said to justify the assumption of collapse, since they show that the experimental results do not require it, but follow directly from a complete analysis of the Schrödinger equation.

We briefly note that Pegg (1991) has used the collapse analysis of these experiments to provide an answer for Squires (1990), who had inquired how quickly collapse of wavefunction may take place. By examining the possible ranges of the various characteristic times of the atomic V-configuration consistent with occurrence of the interrupted fluorescence phenomenon, Pegg was able to produce a value of $10^{-5}$ or $10^{-6}$ s for time of collapse. However, since our view is that wavefunction collapse in the sense of von Neumann is not taking place in these experiments, we do not feel that Pegg's considerations are relevant to Squires' question.

## 5. Conclusions

In this paper, a variety of negative-result experiments have been discussed. In particular it is shown that the neutral kaon regeneration phenomenon is an example of such an experiment. For the various types of experiment, it has been demonstrated that it is not necessary to assume a collapse of the state vector. In particular, the phenomenon of interrupted fluorescence may be explained by taking into account the full state of the system, including the emitted photons.

## Acknowledgments

One of the authors (DH) gratefully acknowledges the Fellowship of the Commission of the European Communities, and the support provided by the Department of Science and Technology (Government of India) for his research work. We would like to thank Professor E Squires for useful discussions.

## References

Ballentine L E 1970 *Rev. Mod. Phys.* **42** 358
—— 1991 *Phys. Rev.* A **43** 5165
Bell J S 1981 *Quantum Gravity 2* ed C Isham, R Penrose and D Sciama (Oxford: Clarendon) pp 611–37
Bohm D and Hiley B J 1987 *Phys. Rep.* **144** 323
Chiu C B, Sudarshan E C G and Misra B 1977 *Phys. Rev.* B **16** 520
Cook R J and Kimble H J 1985 *Phys. Rev. Lett.* **54** 1023
Dehmelt H 1975 *Bull. Am. Phys. Soc.* **20** 60
d'Espagnat B 1976 *Conceptual Foundations of Quantum Mechanics* (Reading, MA: Benjamin)
Dicke R H 1981 *Am. J. Phys.* **49** 925
—— 1986 *Found. Phys.* **16** 107
Epstein P S 1945 *Am. J. Phys.* **13** 127
Feynman R P, Leighton R B and Sands M 1965 *The Feynman Lectures on Physics* vol III (Reading, MA: Addison-Wesley) p 11–5
Gell-Mann M and Pais A 1955 *Phys. Rev.* **97** 1387
Home D and Whitaker M A B 1986 *J. Phys. A: Math. Gen.* **19** 1847
—— 1992a *Phys. Rep.* **210** 223
—— 1992b *J. Phys. A: Math. Gen.* **25** 657
Itano W M, Heinzen D J, Bollinger J J and Wineland D J 1990a *Phys. Rev.* A **41** 2295
—— 1990b *Phys. Rev.* A **43** 5165
Jammer M 1974 *The Philosophy of Quantum Mechanics* (New York: Wiley) p 495
Pegg D T 1991 *Phys. Lett.* **153A** 263
Pegg D T and Knight P L 1988a *Phys. Rev.* A **37** 4303
—— 1988b *J. Phys. D: Appl. Phys.* **21** 5128
Peres A 1980 *Am. J. Phys.* **48** 931
Peres A and Ron A 1990 *Phys. Rev.* A **42** 5720
Petrovsky T, Tasaki S and Prigogine I 1990 *Phys. Lett.* **151A** 109
Porrati M and Putterman S 1987 *Phys. Rev.* A **36** 929
Renninger M 1953 *Z. Phys.* **136** 251
—— 1960 *Z. Phys.* **158** 417
Schenzle A and Brewer R G 1986 *Phys. Rev.* A **34** 3127
Scully M O, Shea R and McCullen J D 1978 *Phys. Rep.* **43** 485
Squires E J 1990 *Phys. Lett.* **148A** 381
von Neumann J 1955 *Mathematical Methods of Quantum Mechanics* (Princeton, NJ: Princeton University Press)
Whitaker M A B 1989 *Eur. J. Phys.* **10** 71